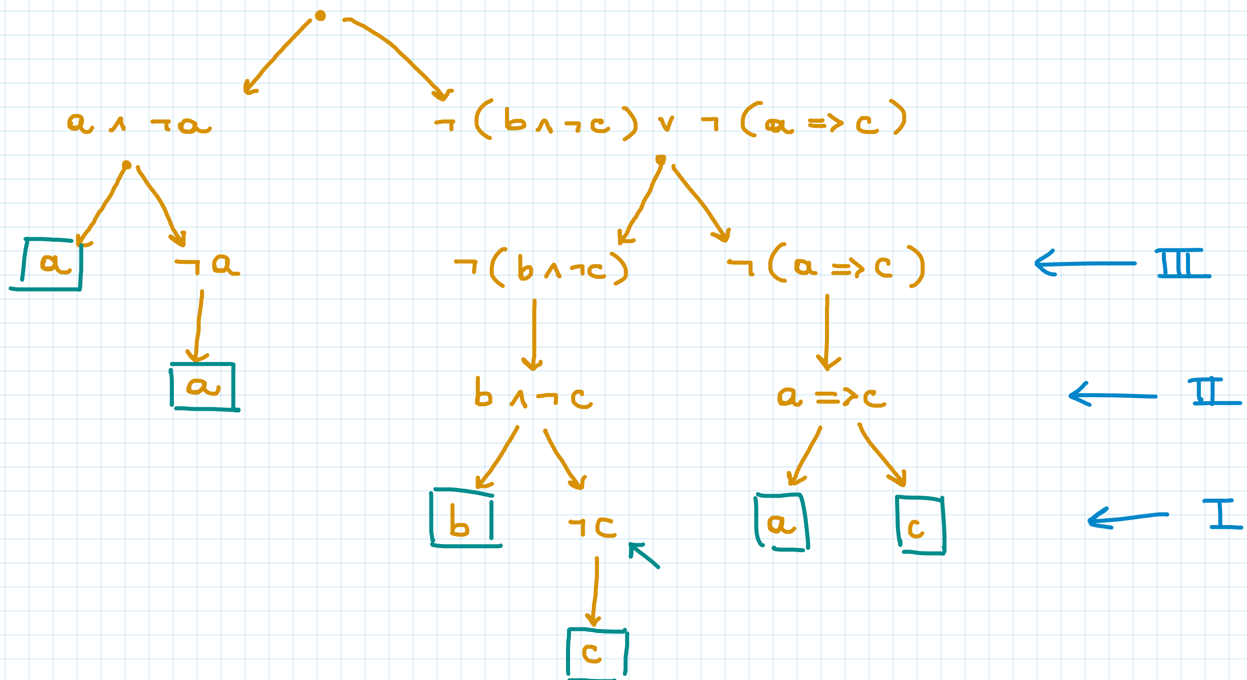
PROVA del 02·07·2024

ES 4

$$\models (a \land \neg a) \boxed{\Rightarrow} \neg(b \land \neg c) \lor \neg(a \Rightarrow c)$$



$$\forall v : \quad v\Big((a \land \neg a) \boxed{\Rightarrow} \neg(b \land \neg c) \lor \neg(a \Rightarrow c)\Big) = 1$$

$$\boxed{v(p \Rightarrow q) = 1 \iff v(p) \leq v(q)}$$ Teoria

$$v(\underbrace{a \land \neg a}_{①}) \overset{?}{\leq} v(\underbrace{\neg(b \land \neg c) \lor \neg(a \Rightarrow c)}_{②})$$

① : $\quad v(a \land \neg a) = \min(v(a), v(\neg a)) = \min(v(a), 1 - v(a))$

$v(\neg p) = 1 - v(p)$

$v(p \land q) = \min(v(p), v(q))$

*

OPPURE

** $\rightarrow v(p \land q) = v(p) \cdot v(q)$

$$= v(a) \cdot v(\neg a) = v(a)(1 - v(a)) =$$

$$= v(a) - v(a) \cdot v(a)$$

$*$ Se $v(a) = 0$, allora $\min(0, 1-0) = 0$

Se $v(a) = 1$, allora $\min(1, 1-1) = \min(1, 0) = 0$

Qualsiasi sia il valore di verità di a, la formula $a \wedge \neg a$ è valutata sempre a 0:

$$\forall v : v(\underbrace{a \wedge \neg a}_{1}) = 0$$

$v(1) \leq v(2)$? è VERO

perchè $0 \leq v(2) \begin{cases} \nearrow 0 \\ \searrow 1 \end{cases}$ $\square$

$**$ $v(1) = v(a) - v(a) \cdot v(a) = v(a) - v(a) = 0$

$\uparrow$

$v(a) \cdot v(a) = v(a)$

$p \wedge p \equiv p$ IDEMPOTENZA

Perciò $\forall v : v(1) = 0$, da cui $v(1) \leq v(2)$ È VERA

perchè $0 \leq v(2) \begin{cases} \nearrow 0 \\ \searrow 1 \end{cases}$ $\square$

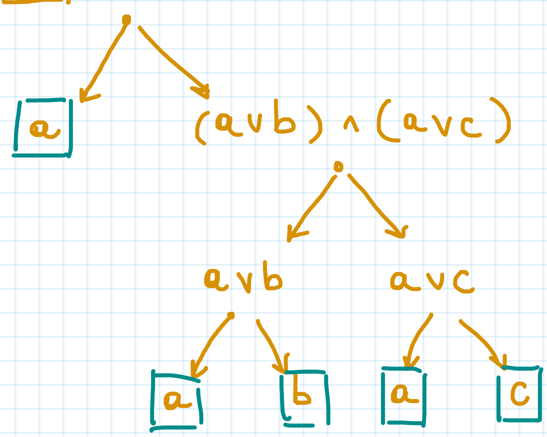
TAVOLA di VERITÀ:

$$\underbrace{\neg}_{3} (\underbrace{b \wedge \underbrace{\neg c}_{1}}_{2}) \boxed{\vee} \underbrace{\neg}_{3} (\underbrace{a \Rightarrow c}_{2})$$

a	b	c	$\neg c$	$b \wedge \neg c$	$a \Rightarrow c$	$\neg(b \wedge \neg c)$	$\neg(a \Rightarrow c)$	$\neg(b \wedge \neg c) \vee \neg(a \Rightarrow b)$
0	0	0	1	0	1	1	0	1
0	0	1	0	0	1	1	0	1
0	1	0	1	1	1	0	0	0
0	1	1	0	0	1	1	0	1
1	0	0	1	0	0	1	1	1
1	0	1	0	0	1	1	0	1
1	1	0	1	1	0	0	1	1
1	1	1	0	0	1	1	0	1

ES 5.

Dim $\vdash a \boxed{\Rightarrow} (a \vee b) \wedge (a \vee c)$



$\longrightarrow$

$$\cfrac{\cfrac{\cfrac{[a]}{a \vee b}\,(I\vee_1)\;{}^{2} \qquad \cfrac{[a]}{a \vee c}\,(I\vee_1)\;{}^{3}}{(a\vee b)\wedge(a\vee c)}\,(I\wedge)\;{}^{4}}{a \Rightarrow (a\vee b)\wedge(a\vee c)}\,(I\Rightarrow)\,[a]\;{}^{5}$$

0. Se a è FALSA, allora $a \Rightarrow (a\vee b)\wedge(a\vee c)$ è sempre VERA
1. Supponiamo che a sia vera
2. Da 1. ottengo $a \vee b$ $(I\vee_1)$
3. Da 1 ottengo $a \vee c$ $(I\vee_1)$
4. Da 2.+3. ottengo $(a\vee b)\wedge(a\vee c)$ con $(I\wedge)$
5. Da 1.+4. ottengo $a\Rightarrow(a\vee b)\wedge(a\vee c)$ con $(I\Rightarrow)$

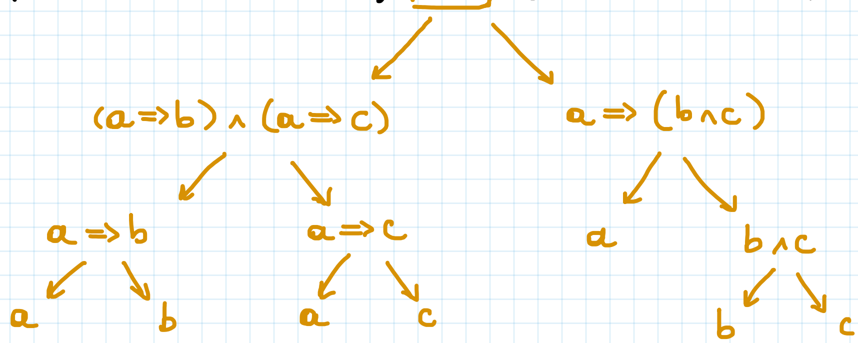
ES 6 $\quad FV(P) \quad P \in FBF$

1. Se $P = \bot$, allora $FV(\bot) = \emptyset$
2. Se $P = P(t_1, t_2, \dots, t_n)$, allora $FV(P(t_1, t_2, \dots, t_n)) =$
$= FV(t_1) \cup FV(t_2) \cup \dots \cup FV(t_n)$
3. Se $P = \neg P_1$, allora $FV(\neg P_1) = FV(P_1)$
4. Se $P = \begin{array}{l} P_1 \wedge P_2 \\ P_1 \vee P_2 \\ P_1 \Rightarrow P_2 \end{array}$, allora $FV(P) = FV(P_1) \cup FV(P_2)$
5. Se $P = \begin{array}{l} (\forall x : P_1) \\ (\exists x : P_1) \end{array}$, allora $FV(P) = FV(P_1) \setminus \{x\}$

PROVA del 06·02·2024

ES 5 :

$$\vdash (a \Rightarrow b) \wedge (a \Rightarrow c) \boxed{\Rightarrow} (a \Rightarrow (b \wedge c))$$



$$* \quad \dfrac{p \wedge q}{p} \qquad ** \quad \dfrac{p \wedge q}{q}$$

$$\dfrac{\dfrac{\dfrac{\dfrac{\boxed{[a]} \quad \dfrac{\boxed{[(a\Rightarrow b)\wedge(a\Rightarrow c)]}}{a\Rightarrow b}\,*}{b}\,(MP) \qquad \dfrac{\boxed{[a]} \quad \dfrac{\boxed{[(a\Rightarrow b)\wedge(a\Rightarrow c)]}}{a\Rightarrow c}\,**}{c}\,(MP)}{b\wedge c}\,(I\wedge)}{\boxed{a}\Rightarrow(b\wedge c)}\,(I\Rightarrow)\,\boxed{[a]}}{\underbrace{(a\Rightarrow b)\wedge(a\Rightarrow c)} \Rightarrow \underbrace{(a\Rightarrow(b\wedge c))}}\,(I\Rightarrow)\,\boxed{[(a\Rightarrow b)\wedge(a\Rightarrow c)]}$$

questa la dovrò trovare SOPRA!

Se NON avessi trovato $(a\Rightarrow b)\wedge(a\Rightarrow c)$.

$$\dfrac{\dfrac{\dfrac{\dfrac{[a] \quad a\Rightarrow b}{b} \qquad \dfrac{[a] \quad a\Rightarrow c}{c}}{b\wedge c}}{a\Rightarrow(b\wedge c)}\,(I\Rightarrow)[a]}{(a\Rightarrow b)\wedge(a\Rightarrow c)\Rightarrow(a\Rightarrow(b\wedge c))}$$

NON le posso cancellare

avrei dimostrato:

$$a\Rightarrow b,\ a\Rightarrow c \vdash (a\Rightarrow b)\wedge(a\Rightarrow c)\Rightarrow(a\Rightarrow(b\wedge c))$$

IPOTESI